Statistical and fractal analysis of the fractures in Derakht-e-Bid Tonalite, West of Mashhad, North-East Iran

Hossein Alizadeh*, Behnam Rahimi[1], Mohsen Fardi Golyan[2]

1Department of Geology, Faculty of Science, Ferdowsi University of Mashhad, Iran

2Department of Geology, Faculty of Science, Islamic Azad University, Mashhad Unit, Iran

*Faculty of Earth Sciences , Damghan University of Basic Sciences, Damghan, Iran
Email address: alizade.hossein@gmail.com

Abstract
The Derakht-e-Bid tonalit, in the north slope of Binaloud Mountain that are exposure among Paleo-Tethys remnants display the structural fractures and our aim is analysis of statistical and fractal nature of them. Based on field observation and contour plots four fractures set revealed. According to kinematic analysis, three set of fractures identified as outcrop scaled faults because of right lateral strike-slip separation. Parameters like length and spacing measured for all fractures set and maximum displacement (d_{max}) and its relation with length calculated only for faults. These factors help us in order to determine whether the spatial distribution of fractures is fitted with power law or not. Based on fracture map whether or not the fracture geometry is self similar, was examined by using a box counting method. The fractal dimension in the Derakhte-Bid tonalite varied from 1.41 to 1.66. The results prove fracture array in this region fitted with power law and seem they have fractal nature

1. Introduction
The elastic properties of earth materials are strongly affected by the spatial distribution of fractures. Recent researches reveal, the fractal or multi-fractal models can be used for describing the spatial distribution of faults and fractures array (King, 1983; Poulimenos, 2000; Babadagli, 2002). It has been the subject of several studies trying to connect scaling laws with characteristic parameters such as length, displacement, spacing and clustering (Turcotte, 1992; Ghosh and Daemen, 1993). Self-similarity is the most striking property of isotropic fractals , in which each piece of a shape is geometrically similar to the whole (physical self-similarity) or its statistical moments change as a power of scale upon multi-scale sampling (statistical self-similarity) (Mandelbrot, 1983; Turner et al, 1998). Computer simulations of synthetic fracture sets have also confirmed the existence of such power laws (Cowie and Scholz, 1992; Cowie et al, 1995).
In this research, the geometrical, statistical and fractal properties of fracture array in Derakht-e-Bid tonalit, west of Mashhad have been studied. Our data obtained from surface exposure of Derakht-e-Bid tonalit that divided into several subareas. In each subarea the fracture sets are recognized and geometrical parameters such as orientation, length, spacing, displacement and fracture density were measured for each set, along scan lines. Moreover we prepared a fracture map for each subarea and calculated the 2D fractal dimension by using box-counting method and also examine the self similarity structure.

2. Geological Setting:

The study area is located in northeastern Iran, west of Mashhad and lies between longitude 59° 21′ E to 59° 26′ E and latitude 36° 19′ N to 36° 23′ N (Fig 1a). The tonalite of Derakhte-e-Bid is exposed among Paleo-Tethys remnants, on the northern slopes of the Binaloud mountains. The Binaloud mountains are part of Paleo-Tethys remnants (meta- Ophiolite and meta-Flysch) which were intruded by tonalite, granodiorite, leucogranite and Pegmatite dikes at different episodes (Triassic to Cretaceous time) (Karimpour et al, 2006).

The Paleo-Tethys remnants are strongly affected by structural deformation at different times. Structures which are formed before the middle Jurassic are related to Cimmerian orogeny, whereas those formed after Late Cretaceous are related to Alpine structures (Alavi, 1991). Field observations show several sets of fractures in Derakht-e-Bid tonalite, each set has a specific characterization, so we divided them in separate cluster.

3. Clustering of fractures in Derakhte-e-Bid tonalite

Fracture orientation data for each subarea are presented as contour plots in the lower-hemisphere equal area (Fig. 1a, b). To ensure statistically robust mean fracture orientations, we measured 30-40 fractures orientations for each fracture set. We can deduced four fractures set in Detrakht-e-Bid tonalite From contour plots and outcrop observations that majority of them have striations. These striations formed along tectonic deformations on the fracture surfaces. According to Pollard and Aydin (1988)and Segall and Pollared (1983), one of these fracture sets (Set IV) is defined as joints because of displacement normal to fracture surfaces and other fracture sets (Set I, II, III) are regarded as right lateral strike-slip faults and maximum separation is about 70 cm.

Among three fault sets, set 1 is most important, because this set could be followed, in plan-view for several meters according to the size of the exposure but sets 2 and 3 are cross and splay fractures of set 1, respectively. In contrast to set 1, set 2 and 3 have small length and separation.

4. Statistical analysis of fracture parameters

4-1. Fractures (or Faults) Trace Length (L) and Maximum Displacement (d_{max}) Distribution

It is generally accepted that many populations of fractures (or faults) have power-law cumulative frequency distributions (Scholz and Cowie, 1990; Yielding et al, 1996). That is the number of fractures (N) of length greater than or equal to L is given by:

$N=\alpha L^{-c}$

Where α is a constant and c is the power law exponent. Ideally, when plotted on Log-Log axes, such distributions appear linear with slope *-c*.

The fault length data (Fig. 2a) show a poor fit to a power law distribution ($C\approx-1.2$). Whilst the departure from a power law can be in part due to censoring of fault lengths due to unexposed tip, the data are best described by a log-normal distribution with median length ~40m.

The maximum displacement data (Fig. 2b) can be fitted by power law with a relatively high value ($C\approx-0.94$) of the power law exponent. Alternatively a log-normal distribution with median ~80mm could also fit the data.

4-2. Maximum Displacement (d_{max}) Versus Fault Length (L) Relationships

The general expression of the relationships between the maximum cumulative displacement on a fault (d_{max}) and the maximum linear dimension of the fault is given as:

$d_{max} = cL^n$

where the value of c is dependent on rock properties (Cowie and Scholz, 1992). The value of exponent, n, is important as n=1 indicates a linear scaling law (i.e., self similarity). Maximum displacement and fault length plots (Fig. 2c) of strike-slip faults data are scattered. The power-law curve fit ($Y=8.71x^{0.85}$, $R^2 \approx 0.9$) is characterized by a high correlation coefficient and a power law exponent of ~0.9, indicating that displacement across faults correlates in an approximately linear fashion with fault length, similar to other data sets analyzed in the literature. Many recent researches have suggested that such faults are scale-invariant and have a constant d_{max}/L ratio (Cowie ans Scholz, 1992).

4-3. Analysis of the Fracture Spacing

The spacing is defined as the spacing between a pair of immediately adjacent fractures of same set, measured along a scan line. Mean spacing of joints and outcrop scale faults varies significantly between outcrops at different subarea. Fracture spacing histograms reveal important aspects of the fracturing process, such as the degree of fracture set development and the influence of flaws on fracture distributions (Rives et al, 1992). Histograms must contain large quantities of data to yield meaningful results, therefore data from all subareas combined together for each set.

According to must publications the relative frequency of fracture spacing can be described by different distribution laws. Log-normal, normal, exponential, gamma and weibull distributions were fitted to the spacing frequency histograms for both joint and fault sets. The data set show a reasonable fit for the negative exponential and log-normal distribution for joints and faults, respectively (Fig. 3a, b). Rives et al (1992) concluded that fracture spacing distributions vary along different stage of fracture development, The spacing distribution is negative exponential at a stage with only few fractures, log-normal at intermediate fracture density, and tends toward normal at high fracture density.

5. Fractal Analysis of the Fracture arrays

5-1. Box counting Analysis of Fracture Maps

A fractal is the general concept for self-similarity introduced by Mandelbrot (1983), and is a powerful tool to characterize the geometry that has a self-similar structure. Self-similarity manifests itself in a power law. The geometry of fractures (or faults) systems seems to be complex and hard to describe, this complexity may be treated by a fractal approach. The most popular technique of analyzing the fractal analysis of fracture system is the box-counting method which allow one to draw log-log plots of the number of boxes containing the structures (Ns) against the size of the measuring grid (s), or its reciprocal (1/s), and to derive box counting curves that typically show slop values between 1 and 2 (Mandelbrot, 1983).

According to this technique, consider a fracture system is enclosed in a grid of N_0 square region with a side length r_0. In the next steps, divide the square region into (r_0/r^2) square boxes of side length, r. Let N(r) be the number of boxes that the fractures line enters. If a fracture system has a self similar structure, we get the following relation:

$$N(r) \sim (r_0 / r)^D \sim r^{-D}$$

where D is a fractal dimension. Practically, N(r) is plotted against r on double logarithmic scale; the graph is almost linear with slop –D.

We use the box dimension method in the present work as a convenient fractal characteristic. First we take photographs from each subareas then each image analyzed with the shareware "Fractal Analysis" developed by Tolson (2003). The box dimensions for all subarea were calculated and are shown in table 1. The sampled fracture traces in the studied subareas have box dimensions between 1.41 and 1.66 (Table1). Bonnet et al (2001), discussed 87 published fractal dimensions obtained from two dimensional analyses, which vary from 1 to 2, the most frequent values clustering around 1.5 and 2. Values equal or very close to 2 probably correspond to non-fractal objects.

6. Conclusions

In this work, we analysis the statistical parameters and fractal dimension of fractures in Derakht-e-Bid tonalite to prove the self-similarity of fractures array. The following conclusions can be drawn:

- The fractures length data show a poor fit to a power law distribution ($C \approx -1.2$) because of unexposed tip. The maximum displacement data (d_{max}) show a better fit to a power law distribution ($C \approx -0.94$).
- Plot data of maximum displacement and faults length are scattered. The power-law curve fit shows a high correlation coefficient and a power law exponent of ~0.9, indicating that displacement across faults correlates in an approximately linear fashion with fault length, similar to other data sets analyzed in the literature.
- The spacing data show a reasonable fit for the negative exponential and log-normal distribution for joints and faults, respectively that concluded the joints are at the first stages of development whereas faults determine the final stages development.
- The sampled fracture traces in the studied subareas have box dimensions between 1.41 and 1.66 that suggested the fractures array in Derakhte-Bid tonalite are candidate for fractal theory. As a result spatial distributions of fractures in Derakhte-Bid Tonalite are self similar and scale invariant because of best fitted with power law distribution.

7. References

1. King, G., 1983, The accommodation of large strains in the Upper lithosphere of the earth and other solids by self-similar fault systems: The geometrical origin of b value: Pure and Applied Geophysics, 121, p. 761-815.
2. Poulimenos, G., 2000, Scaling properties of normal fault populations in the Western Corinth Networks, Greece: Implication for fault growth in large strain setting: Journal of structural geology, 22, p. 307-322.
3. Babadagli, T., 2002, Scaling method to determine the fractal nature of 2-D fracture networks: Math. Geol, 34, p. 647-670.
4. Trucotte, D. L., 1992, Fractal and chaos in geology and geophysics: Cambridge university press, Cambridge.

5. Ghosh, A and Daemon, J. J. K., 1993, Fractal characteristics of rock discontinuities: Eng. Geol, 34, p. 1-9.
6. Mandelbrot, B., 1983, The fractal geometry of nature: Freeman and company, New York.
7. Turner, M. J., Blackledge, J. M., and Andrews, P. R., 1998, Fractal geometry in digital imaging: Academic press, University printing House, Cambridge.
8. Cowie, P. A., Scholz. C. H., 1992, Displacement-length scaling relationship for fault: data synthesis and discussion: Journal of structural geology, 10, p. 1149-1156.
9. Cowie, P. A., Sornette, D., and Vanneste, C., 1995, Multifractal scaling properties of growing fault population: Geophys. J. Int., 122, p. 457-469.
10. Karimpour, M. H., Farmer, L., Ashouri, C., and Saadat. S., 2006, Major, trace and REE geochemistry of Paleo-Tethys collision-related Granitoids from Mashhad, Iran: Journal of sciences, Islamic Republic of Iran, 17(2), p. 127-145.
11. Alavi, M., 1991, Sedimentary and structural characteristics of the Paleo-Tethys remnants in northeastern Iran: Geological Society of America Bulletin, 103, p. 983-992.
12. Pollard, D. D., Aydin, A., 1988, Progress in understanding jointing over the past century, Geological Society of America Bulletin, V. 100, p. 1181-1204.
13. Segall, P., Pollard, D. D., 1983, Joint formation in granitic rock of the Sierra Nevada: Geological Society of America Bulletin, 94, p. 563-575.
14. Scholz, C. H., and Cowie, P., 1990, Determination of total strain from faulting using slip measurements: Nature, 346, p. 837-838.
15. Yielding, G., Needham, T., Jones, H., 1996, Sampling of fault populations using sub-surface data: a review: Journal of structural geology, 18, p. 135-146.
16. Rives, T., et al., 1992, Joint spacing: analogue and numerical simulations: Journal of tructural geology, 14, p. 925-937.
17. Tolson, G., Nieto-Samanieto, A. F., and Alianiz-Alvarez, S. A ., 2003, Fractal analysis: University of Nacional Autónoma de México, Institute de Geologia, available from World Wide Web: http://Geologia.igeolcu.unam.mx/Tolson/Gtolson.htm.
18. Bonnet, E., Bour, O., Odling, N. E., Davy, P., Main, I., Cowie, P., Berkowitz, B., 2001, Scaling of fracture systems in geological media: Rev. Geophys, 39, p. 347-383.

Tables and Figures

Table 1: Box dimension in each subarea were calculated with Fractal analysis Program from fracture maps.

No. Subarea	Box. Dimension
1	1.46
2	1.38
3	1.41
4	1.47
5	1.66
6	1.58
7	1.62
8	1.63

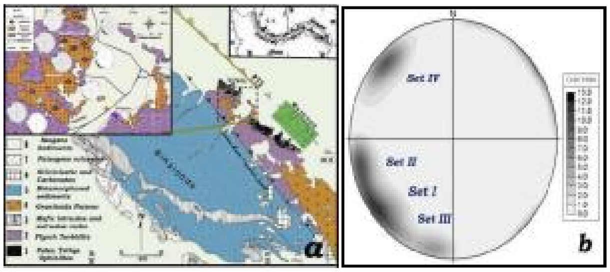

Figure 1- A. Geological map of the study area showing contour plots in each subarea. B. Contour plot of fracture orientation data from all subarea in the lower hemisphere equal area showing four fracture sets.

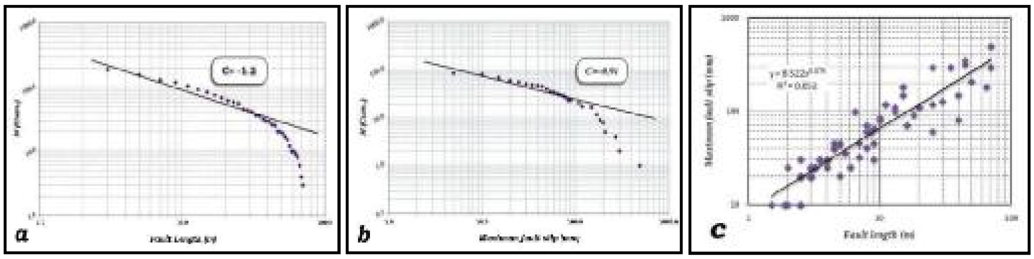

Figure 2. (a) Logaritmic plot of fault length showing log-normal distribution due to censoring. (b) Logaritmic plot of maximum displacement of faults. *C* denote slopes for log-log plots. (c) Logaritmic plot of maximum displacement versus fault length.

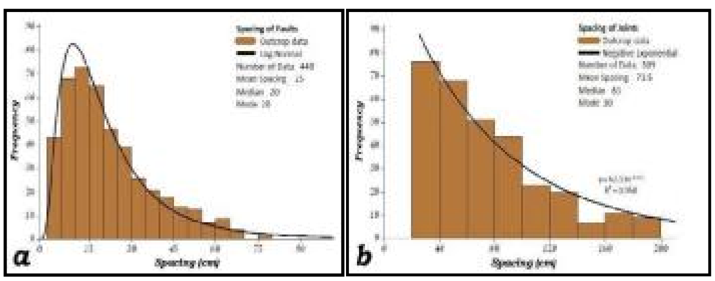

Figure 3. Histogram of fracture Spacing. (a) Spacing of fault with log-normal distribution (b) Spacing of joints with negative exponential distribution